6.4 FACTOR OF SAFETY

As a result of tension and compression tests we obtain basic data on the mechanical properties of a material. Let us now see how the results so obtained can be used in machine design.

As already stated in Chapter 4, the fundamental and most commonly used method of analysis is the method based on stresses. According to this method the design is based on the maximum stress developed at some point of a loaded structure. This stress is called the maximum working stress. It must not exceed a certain value, characteristic of a given material and the service conditions of the structure.

Design based on stresses uses a relation of the form

$$\sigma_{\text{max}} = \frac{S_L}{N}$$

(6.1)

where $S_L$ is a certain limiting stress for a given material, and $N$ is a number greater than unity called the ‘factor of safety’. The dimensions of a structure are usually already known and assigned, for example, from service or technological considerations. In this case the value of $\sigma_{\text{max}}$ is calculated to determine the existing factor of safety

$$N = \frac{S_L}{\sigma_{\text{max}}}$$

(6.2)

If this factor of safety satisfies the designer, it is concluded that the design is acceptable.

When a structure is in the design stage and some characteristic dimensions are to be assigned directly from strength requirements, the magnitude of $N$ is prescribed beforehand. The required dimension is obtained from a relation of the form

$$\sigma_{\text{max}} \leq \frac{S_L}{N}$$

(6.3)

This quantity is called the ‘allowable stress’. It remains to decide what stress is to be taken as a limiting stress $S_L$ and how to assign $N$.

In order to avoid appreciable permanent deformation of a functioning structure it is customary in machine design to take the yield strength as $S_L$ for ductile materials. Then the maximum working stress is $S_y/N$ (Figure 6.17). Here the factor of safety is denoted by $N$ and is called the factor of safety with respect to yielding. For brittle materials and in some cases for moderately ductile materials, $S_L$ is taken to be the ultimate tensile strength $S_u$. We then

![Figure 6.17 Definition of the safety factor](image)
obtain
\[ N_u = \frac{S_u}{S_{\text{max}}} \]  \hspace{1cm} (6.4)

where \( N_u \) is the factor of safety with respect to fracture.

As stated in Chapter 4, the design is based on the limiting load. It is possible to introduce similarly the factor of safety based on the limiting load:
\[ N = \frac{P_L}{P_W} \]  \hspace{1cm} (6.5)

where \( P_L \) and \( P_W \) are the limiting and working loads. In design based on stiffness
\[ N = \frac{\delta_L}{\delta_W} \]  \hspace{1cm} (6.6)

where \( \delta_L \) and \( \delta_W \) are the limiting and working displacements, respectively.

The choice of \( N \) is made in two ways: (a) analytically, based on reliability analysis as shown in Section 6.5; (b) empirically, as is shown below.

The factor of safety assigned is based on experience with the specific service conditions for the structure being designed. The factor \( N \) is virtually determined by past practice and by the state of the art at the particular moment. Each field of engineering has its own traditions, requirements, methods and finally, specificity of designs, and the factor of safety is assigned accordingly. Thus, for example, in the designing of stationary engineering structures intended for prolonged service, the factors of safety are rather large \( (N = 2-5) \). In aircraft engineering where severe weight restrictions are imposed on the structures, they are in the range 1.5 to 2. In view of the high reliability requirements it has become the practice in this field to conduct obligatory static tests on individual components and complete aircraft for direct determination of the limiting loads.

The choice of the factor of safety depends on the methods of stress analysis, the degree of accuracy of these methods, and the graveness of consequences that the failure of a part may entail. The factor of safety depends also on the properties of the material. In the case of a ductile material, the factor of safety with respect to yielding may be lower than for a part made of brittle material. This is quite evident since a brittle material is more sensitive to accidental damage and unexpected manufacturing defects. Moreover, any accidental increase of stresses may cause only small permanent deformations in a ductile material, whereas for a brittle material this may result in failure. The proper choice of the factor of safety depends to a considerable extent on the judgement, experience and ingenuity of the
analyst and the designer, but also on the degree of uncertainty on material properties, methods of analysis and service condition.

The safety factor is not, as commonly thought, audge factor to give us a margin of safety, that is a secure higher strength than the maximum expected load. It is a factor which is characteristic of our uncertainty on the material properties, methods of analysis and service conditions. If the above factors were known exactly, then the safety factor would be 1.

The differential method for the estimation of the factor of safety assumes this factor as a product of several subfactors:

\[ N = N_1 N_2 N_3 N_4 \ldots \]  

(6.7)

where:

- \( N_1 \) reflects the reliability with which design loads can be determined. It takes values between 1 and 1.5. \( N_1 = 1 \), when rated loads are determined with unquestionable accuracy, or where safety devices protect the machine from overloading, for example pressure vessels with relief valves; \( N_1 = 1.5 \), when the load is determined from questionable data, for example wind generators.

- \( N_2 \) reflects the reliability of the material properties. It is taken to be 1.2 to 1.5 for rolled or forged steel and 1.5 to 2.5 for cast iron and brittle material partly owing to easier inspection of the rolled steel and smaller effect of imperfection on ductile materials.

- \( N_3 \) depends on the consequences of a failure. It is taken to be 1 when failure will not affect anything else, or \( \geq 1.5 \) when failure results in total loss of the machine, environmental damage, or danger to operators.

- \( N_4 \) refers to starting and accidental overloads, if the calculations are based on the rated load of the machine, to frequent starts, to operation with or without shocks, etc. This subfactor, also called ‘service factor’, depends on the particular application and can sometimes reach high values. \( N_4 \) can be taken up to a value of 2 for smooth operation and low starting torque, to 5 for rough operation, high starting torque and frequent start-ups. Of course it is taken to be 1 when the exact operating loads have been used in the calculations. Design is based on the yield strength.

As a guide only, and if design is based on both the yield and fatigue strengths, \( N_4 \) is taken (see K-H. Decker, *Maschinenelemente*, 1973) to be:

1.3 to 1.5 for static loads, turbines and electric machines;
1.4 to 1.6 for reciprocating machines and machine tools;
1.5 to 1.8 for punching and pressing machines; and
1.6 to 2.0 for impact machines and steel mills.