Heat Transfer Summary (from Chapter 13 of online notes)

Mathematical terms:

Scalar- a quantity with no subscripts, i.e. no directional dependence, just a magnitude.
Vector- a quantity with one subscript that has a magnitude and a directional dependence.
Tensor-a quantity with multiple subscripts that transforms to other coordinates using direction cosines.
A vector is a first order tensor. A second order tensor has two subscripts (similar to rectangular matrices).

There are three types of heat transfer: internal conduction (easy), surface convection and known surface heat flows (less easy) and surface radiation (difficult). Radiation studies are nonlinear and require the use of the absolute temperature scale (Kelvin). All other thermal studies can use any convient temperature scale (such as F or C).

Thermal studies involve the scalar temperature, *T,* the heat flux vector (per unit area) $\vec{q}$, and the scalar heart flow (area integral of normal heat flux). The governing differential equation for the temperature in internal conduction in one-dimension is

$$\frac{\partial}{\partial x}\left(k_{xx}\frac{\partial T}{\partial x}\right) + Q(x) = \rho c_p \frac{\partial T}{\partial t}.$$

Here, *T* is the temperature, *x*, the position, k_{xx}, is the thermal conductivity material property in the x-direction, Q is the internal rate of heat generation per unit length (also called heat power), *t* denotes time, and ρc_p are the mass density and specific heat material properties, respectively. The essential boundary conditions are specified temperature values. Most materials have the same conductivity in all directions (are isotropic, so $k_{xx} = k_{xy} = k_{yy} = k$). Otherwise, the thermal conductivity is a second order tensor material property. Fourier's Law defines the heat flux vector as the negative of the conductivity times the temperature gradient: $\vec{q} = -k\vec{\nabla}T$. In metric units it is displayed as W/m^2.

For pure steady state conduction (*Q=0*, $\partial/\partial t = 0$) with constant conductivity the equation reduces to $k_{xx}\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = 0$, and the temperature is linear between two known end values, $T(x) = T_0 + (T_L - T_0)\,x/L$. Then the heat flux becomes $q_x = -k_{xx}\,\partial T/\partial x$. The heat flux is usually plotted as a vector quantity, and as contours of its magbitude.

Surface convection involves a surface condition known as the convection coefficient. It varies greatly and often requires a formal heat transfer course (mech 481) to establish its value. When air is the surrounding medium, thr typical value is $h = 5 - 25\ W/m^2K$, and the typical water convection values are $h = 500 - 1{,}000\ W/m^2K$. The normal convection heat flux per unit area is $q_h = h(T - T_m)$, where T is the (unknown) surface temperature and T_m is the known temperature of the surrounding convecting medium.

Radiation surface heat flux is proportional to the difference in the fourth power of the absolute temperatures: $q_r = \varepsilon\sigma(T^4 - T_m^4)$.

Heat flow crossing normal to a surface is the integral of the normal heat flux: $H = \int_A \vec{q}\cdot\vec{n}dA$. Heat flow is necessary to maintain a specified temperature boundary condition. Ih a heat flow is specified at a point, then the temperature at that point is computed.