EXAMPLE 4-10

The A-36 steel bar shown in Fig. 4–18 is constrained to just fit between two fixed supports when $T_1 = 60^{\circ}\text{F}$. If the temperature is raised to $T_2 = 120^{\circ}\text{F}$, determine the average normal thermal stress developed in the bar.

SOLUTION

Equilibrium. The free-body diagram of the bar is shown in Fig. 4-18b. Since there is no external load, the force at A is equal but opposite to the force acting at B; that is,

$$+\uparrow \Sigma F_{\nu} = 0; \qquad F_A = F_B = F$$

The problem is statically indeterminate since this force cannot be determined from equilibrium.

Compatibility. Since $\delta_{A/B} = 0$, the thermal displacement δ_T at A that would occur, Fig. 4–18c, is counteracted by the force **F** that would be required to push the bar δ_F back to its original position. The compatibility condition at A becomes

$$(+\uparrow) \qquad \qquad \delta_{A/B} = 0 = \delta_T - \delta_F$$

Applying the thermal and load-displacement relationships, we have

$$0 = \alpha \Delta T L - \frac{FL}{AE}$$

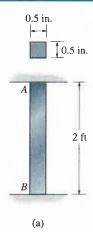
Thus, from the data on the inside back cover,

$$F = \alpha \Delta T A E$$
= [6.60(10⁻⁶)/°F](120°F - 60°F)(0.5 in.)²[29(10³) kip/in²]
= 2.87 kip⁴

From the magnitude of **F**, it should be apparent that changes in temperature can cause large reaction forces in statically indeterminate members

Since F also represents the internal axial force within the bar, the average normal compressive (thermal) stress is thus

$$\sigma = \frac{F}{A} = \frac{2.87 \text{ kip}}{(0.5 \text{ in.})^2} = 11.5 \text{ ksi}$$
 Ans.





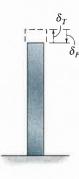


Fig. 4-18

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