



# NOTATION

The symbols most commonly used throughout the book are defined below. When appearing in the text matrices, tensors, and vectors are identified by boldface type.

## Mathematical Symbols

$(\hat{\cdot})$	Based on element gradient
$(\cdot^*)$	Based on nodally continuous gradient
$\{.\}$	Column vector, n by 1
$ \cdot $	Determinant of a matrix
$\Delta^T$	Divergence operator
$\emptyset$	Empty set
$\nabla$	Gradient operator
$\in$	In
$\cap$	Intersection
$[\cdot]^{-1}$	Inverse of a square matrix
$\square$	Non-dimensional parametric space
$\ \cdot\ $	Norm of a matrix or vector
$],.,.[$	Open one-dimensional domain
$[\cdot]^T[\cdot]$	Outer product square matrix, m by m
$\cdot,(\cdot)$	Partial differentiation with respect to $(\cdot)$
$\partial_G, \partial_\Omega$	Partial derivatives in global Cartesian space
$\partial_L, \partial_\square$	Partial derivatives in local parametric space
$\propto$	Proportional to
$[\cdot]$	Rectangular, m by n, or square matrix
$[\cdot]$	Row vector, 1 by m
$\subset$	Subset
$[\cdot]^T$	Transpose of a matrix
$\cup$	Union

## Latin Symbols

$A$	Area
$\mathbf{a}$	Acceleration vector
$a, b, c$	Natural coordinates on $-1$ to $+1$
$(\cdot)^b$	Relating to a boundary domain
$\mathbf{B}$	Differential operator acting on interpolation matrix $\mathbf{H}$ or $\mathbf{N}$

<b>b</b>	Differential operator acting on global interpolation matrix <b>h</b>
$C^n$	Field continuity of degree $n$
<b>C</b>	System source vector
$C^b$	Source vector from a boundary segment
$C^e$	Source vector from an element
<b>D</b>	System degrees of freedom vector
$D$	Differential operator.
$D^b$	Boundary segment degrees of freedom vector
$D^e$	Element degrees of freedom vector
<b>d</b>	Cartesian gradient of <b>H</b>
$d_x$	First row of <b>d</b> , etc. for y, z
<i>d. o. f.</i>	Degree(s) of freedom
$E$	Modulus of elasticity of a material
<b>E</b>	Constitutive law (stress-strain) matrix
$e$	Error
$(.)^e$	Relating to an element domain
<b>F</b>	Resultant force vector
$G$	Shear modulus of a material
<b>G</b>	Geometry interpolation row matrix (usually <b>G</b> = <b>H</b> )
$H^b$	Boundary interpolation row matrix for a scalar
$H^e$	Element interpolation row matrix for a scalar
$h$	Characteristic length. Convection coefficient
<b>h</b>	Global interpolation matrix
$I^e, \mathbf{I}^e$	Integral of a scalar or matrix, respectively, on an element
<b>I</b>	Identity matrix
<b>J</b>	Jacobian matrix of a geometric transformation
<b>K</b>	Stiffness matrix
$k$	Thermal conductivity of a material, or spring stiffness
<b>L</b>	Differential operator
$L$	Length
$L_k$	Baracentric coordinates, $\sum L_k = 1$
<b>M</b>	Mass matrix of the system
$m^e$	Mass matrix, or thermal capacity matrix of an element
$m$	Mass
<b>N</b>	Interpolation matrix for generalized degrees of freedom (often <b>N</b> = <b>H</b> )
<b>n</b>	Unit normal vector
$n_a$	Number of adjacent elements, $NEIGH\_L$
$n_b$	Number of boundary segments, $N\_MIXED + N\_SEG$
$n_c$	Number of constraint equations, $N\_CEQ$
$n_d$	Number of system degrees of freedom ( $n_m \times n_g$ ), $N\_D\_FRE$
$n_e$	Number of elements in the system, $N\_ELEMS$
$n_f$	Maximum number of flux components, $N\_G\_FLUX$
$n_g$	Number of generalized dof per node, $N\_G\_DOF$
$n_i$	Number of element equation index terms ( $n_n \times n_g$ ), $LT\_FREE$
$n_m$	Maximum node number in the system, $MAX\_NP$

$n_n$	Maximum number of nodes per element, $NOD\_PER\_EL$
$n_p$	Dimension of the parametric space, $N\_PARM$
$n_{pe}$	Number of elements in a patch, $L\_IN\_PATCH$
$n_q$	Number of quadrature points, $N\_QP$
$n_r$	Number of rows in the $\mathbf{B}$ matrix, $N\_R\_B$
$n_s$	Dimension of the physical space, $N\_SPACE$
$n_t$	Number of different element types, $N\_L\_TYPE$
$n_x$	Number of element geometry definition nodes, $N\_GEOM$
$\mathbf{P}$	Polynomial row matrix. Reaction vector
$p$	Pressure
$\mathbf{Q}$	Source per unit volume
$\mathbf{Q}^e$	Source per unit volume at element node points
$q$	Source per unit length
$q_n$	Heat flux normal to boundary ( $\mathbf{q}_n = q_n \mathbf{n}$ )
$\mathbf{q}$	Heat flux vector at a point
$\mathbf{R}$	Matrix of position vectors, $\mathbf{R} = [\mathbf{x} \ \mathbf{y} \ \mathbf{z}]$
$R$	Residual error in $\Omega^e$
$r, s, t$	Unit coordinates on 0 to 1
$\mathbf{S}$	Square matrix of the system
$\mathbf{S}^b$	Square matrix from a boundary segment
$\mathbf{S}^e$	Square matrix from an element
$t$	Thickness, time
$\mathbf{T}$	Transformation matrix, or boundary traction matrix
$U$	Strain energy
$\mathbf{u}$	Displacement vector. Velocity vector
$u, v, w$	Components of displacement vector
$V$	Volume
$\mathbf{v}$	Velocity vector
$W$	Mechanical work
$x, y, z$	Cartesian coordinates
$\mathbf{X}$	Body force vector
$\mathbf{x}$	Vector of x-coordinates
$\mathbf{x}^e$	Vector of x-coordinates of the element nodes
$\mathbf{y}$	Vector of y-coordinates
$\mathbf{z}$	Vector of z-coordinates

### Greek Symbols

$\alpha$	Coefficient of thermal expansion
$\beta$	Boolean gather matrix
$\beta^T$	Boolean scatter matrix
$\sum^e \beta^{eT} \mathbf{C}^e$	Column vector assembly process
$\sum^e \beta^{eT} \mathbf{S}^e \beta^e$	Square matrix assembly process
$\Gamma$	Boundary of a domain, $\Omega$
$\Gamma^b$	Segment of the boundary $\Gamma$

$\Gamma^e$	Boundary of an element domain, $\Omega^e$
$\gamma$	Weight per unit volume
$\Delta$	Local derivatives of the interpolation matrix $\mathbf{H}$ or $\mathbf{N}$
$\delta$	Element or boundary segment d.o.f.
$\varepsilon$	Strain or gradient
$\zeta$	Refinement parameter
$\eta$	Allowed percentage error
$\theta$	Temperature, or angle
$\Theta$	Effectivity index
$\lambda$	Direction cosine. Lamé' constant.
$\mu$	Direction cosine. Lamé' constant.
$\nu$	Poisson's ratio for a material
$\Pi$	Total potential energy, $\Pi = U - W$
$\pi$	Mathematical constant 3.14159...
$\rho$	Mass density of a material
$\boldsymbol{\rho}$	Position vector to a point, $\boldsymbol{\rho} = [x, y, z]$
$\boldsymbol{\sigma}$	Flux or stress
$\boldsymbol{\sigma}^*$	Smoothed flux or stress approximation
$\hat{\boldsymbol{\sigma}}$	Finite element flux or stress approximation
$\tau$	Stabilization parameter
$\tau$	Shear stress
$\Phi$	System degrees of freedom vector
$\Phi_k$	k-th unknown
$\phi$	Scalar unknown. Velocity potential
$\psi$	Stream function
$\omega$	Angular velocity
$\Omega$	Domain
$\Omega^e$	Element domain

**Selected Program Notation** (Array sizes follow in parentheses.)

AJ	JACOBIAN MATRIX: (N_SPACE, N_SPACE)
B	GRADIENT VERSUS DOF MATRIX: (N_R_B, LT_FREE)
C	ELEMENT COLUMN MATRIX: (LT_FREE)
CC	COLUMN MATRIX OF SYSTEM EQUATIONS: (N_D_FRE)
COORD	COORDINATES OF ALL NODES ON ELEMENT: (LT_N, N_SPACE)
C_B	BOUNDARY SEGMENT COLUMN MATRIX
D	NODAL PARAMETERS ASSOCIATED WITH AN ELEMENT: (LT_FREE)
DD	SYSTEM LIST OF NODAL PARAMETERS: (N_D_FRE)
DGH	GLOBAL DERIVS OF SCALAR FUNCTIONS H: (N_SPACE, LT_N)
DGV	GLOBAL DERIVS OF VECTOR FUNCTIONS V: (N_SPACE, LT_FREE)
DLG	LOCAL DERIVS OF GEOMETRY FUNCTIONS G: (LT_PARM, LT_GEOM)
DLH	LOCAL DERIVATIVES OF SCALAR FUNCTIONS H: (LT_PARM, LT_N)
E	CONSTITUTIVE MATRIX: (N_R_B, N_R_B)
EL_M	ELEMENT MASS MATRIX: (LT_FREE, LT_FREE)
G	INTERPOLATION FUNCTIONS FOR GEOMETRY: (LT_GEOM)

GLOBAL	GLOBAL DERIV'S OF SCALAR INTERPOLATION FUNCTIONS H
H	INTERPOLATION FUNCTIONS FOR AN ELEMENT SCALAR: (LT_N)
H_INTG	INTEGRAL OF SCALAR INTERPOLATION FUNCTIONS: (LT_N)
H_QP	INTERPOLATION FOR H AT QUADRATURE POINT: (LT_N, LT_QP)
INDEX	SYSTEM DEGREE OF FREEDOM NUMBERS ARRAY: (LT_FREE)
LT	ELEMENT TYPE NUMBER
LT_FREE	NUMBER OF DEGREES OF FREEDOM PER ELEMENT
LT_GEOM	NUMBER OF GEOMETRIC NODES PER ELEMENT
LT_N	MAXIMUM NUMBER OF NODES FOR ELEMENT TYPE
LT_PARM	DIMENSION OF PARAMETRIC SPACE FOR ELEMENT TYPE
LT_QP	NUMBER OF QUADRATURE POINTS FOR ELEMENT TYPE
LT_SHAP	ELEMENT TYPE SHAPE FLAG NUMBER
L_B_N	NUMBER OF NODES ON AN ELEMENT BOUNDARY SEGMENT
L_SHAPE	ELEM SHAPE, 1=LINE 2=TRI 3=QUAD 4=HEX 5=TET 6=WEDGE
L_TYPE	ELEMENT TYPE NUMBER: (L_S_TOT)
MAT_FLO	NUMBER OF REAL MATERIAL PROPERTIES
MAX_NP	NUMBER OF SYSTEM NODES
MISC_FL	NUMBER OF MISC. FLOATING POINT SYSTEM PROPERTIES
MISC_FX	NUMBER OF MISC. FIXED POINT SYSTEM PROPERTIES
M_B_N	NUMBER OF NODES ON A MIXED BC SEGMENT
NODES	NODE INCIDENCES OF ALL ELEMENTS: (L_S_TOT, NOD_PER_EL)
NOD_PER_EL	MAXIMUM NUMBER OF NODES PER ELEMENT
N_BS_FIX	NUMBER OF BOUNDARY SEGMENT INTEGER PROPERTIES
N_BS_FLO	NUMBER OF BOUNDARY SEGMENT REAL PROPERTIES
N_D_FLUX	MAXIMUM NUMBER OF FLUX SEGMENT DOF, = L_B_N * N_G_DOF
N_D_FRE	TOTAL NUMBER OF SYSTEM DEGREES OF FREEDOM
N_ELEMS	NUMBER OF ELEMENTS IN SYSTEM
N_EL_FRE	NUMBER OF DEGREES OF FREEDOM PER ELEMENT
N_GEOM	NUMBER OF GEOMETRY NODES
N_G_DOF	NUMBER OF GENERALIZED PARAMETERS (DOF) PER NODE
N_G_FLUX	NUMBER OF FLUX COMPONENTS PER SEGMENT NODE
N_LP_FIX	NUMBER OF INTEGER ELEMENT PROPERTIES
N_LP_FLO	NUMBER OF FLOATING POINT ELEMENT PROPERTIES
N_MX_FIX	NUMBER OF FIXED POINT MIXED_SEGMENT PROPERTIES
N_MX_FLO	NUMBER OF FLOATING POINT MIXED_SEGMENT PROPERTIES
N_NP_FIX	NUMBER OF FIXED POINT NODAL PROPERTIES
N_NP_FLO	NUMBER OF FLOATING POINT NODAL PROPERTIES
N_PARM	DIMENSION OF PARAMETRIC SPACE
N_QP	MAXIMUM NUMBER OF QUADRATURE POINTS
N_R_B	NUMBER OF ROWS IN B AND E MATRICES
N_SEG	NUMBER OF ELEM BOUNDARY SEGMENTS WITH GIVEN FLUX
N_SPACE	DIMENSION OF SPACE
PT	QUADRATURE COORDINATES: (LT_PARM, LT_QP)
S	ELEMENT SQUARE MATRIX: (LT_FREE, LT_FREE)
SCP_FIT	NUMBER IF TERMS BEING FIT, N_R_B USUALLY

SCP_GEOM	NUMBER OF PATCH GEOMETRY NODES
SCP_H	INTERPOLATION FUNCTIONS FOR PATCH, USUALLY IS H (SCP_N)
SCP_LT	PATCH TYPE NUMBER
SCP_N	NUMBER OF NODES PER PATCH
SCP_PARM	NUMBER OF PARAMETRIC SPACES FOR PATCH
SCP_QP	NUMBER OF QUADRATURE POINTS NEEDED IN A SCP
SCP_SHAP	PATCH SHAPE FLAG NUMBER
SIGMA_HAT	FLUX COMPONENTS AT PT FROM ORIGINAL ELEMENT: (SCP_FIT)
SIGMA_SCP	FLUX COMPONENTS AT PT FROM SMOOTHED SCP: (SCP_FIT)
SS	SQUARE MATRIX OF SYSTEM EQUATIONS: (N_D_FREE, N_D_FREE)
STRAIN	STRAIN OR GRADIENT VECTOR: (N_R_B + 2)
STRAIN_0	INITIAL STRAIN OR GRADIENT VECTOR, IF ANY: (N_R_B)
STRESS	STRESS VECTOR: (N_R_B + 2)
S_B	BOUNDARY SEGMENT SQUARE MATRIX, IF ANY: (LT_FREE, LT_FREE)
V	INTERPOLATION FUNCTIONS FOR VECTORS: (LT_FREE)
WT	QUADRATURE WEIGHTS: (LT_QP)
X	COORDINATES OF ALL SYSTEM NODES: (MAX_NP, N_SPACE)
XYZ	SPACE COORDINATES AT A POINT: (N_SPACE)