417 Summary 2/6/20, Parametric elements

$$u(x) = u(r) = \sum_{j=1}^{n_i} H_j(r)\, u_j^e = [\boldsymbol{H}(r)]\{\boldsymbol{u^e}\} = \{\boldsymbol{u^e}\}^T[\boldsymbol{H}(r)]^T, \ \ for\ x\ in\ \Omega^e$$

$$Q(r) = \sum_{j=1}^{n_n} H_j(r)\, Q_j^e = [\boldsymbol{H}(r)]\{\boldsymbol{Q^e}\} = \{\boldsymbol{Q^e}\}^T[\boldsymbol{H}(r)]^T, \ \ for\ x\ in\ \Omega^e$$

$$x(r) = \sum_{j=1}^{n_n} H_j(r)\, x_j^e = [\boldsymbol{H}(r)]\{\boldsymbol{x^e}\} = \{\boldsymbol{x^e}\}^T[\boldsymbol{H}(r)]^T, \ \ for\ x\ in\ \Omega^e$$

$$dx = \frac{dx}{dr}dr = J(r)dr, and\ \frac{d}{dx} = \frac{d}{dr}\frac{dr}{dx} = \frac{1}{J(r)}\frac{d}{dr}, and\ iff\ x \equiv x_1^e + L^e r\ then\ J(r) = L^e$$

$$\frac{du(r)}{dx} = \frac{du(r)}{dr}\frac{dr}{dx} = \frac{1}{J(r)}\frac{du(r)}{dr} = \frac{1}{J(r)}\frac{d[\boldsymbol{H}(r)]}{dr}\{\boldsymbol{u^e}\} = \frac{1}{J(r)}\left[\frac{d\boldsymbol{H}(r)}{dr}\right]\{\boldsymbol{u^e}\} = \frac{1}{J(r)}\sum_{j=1}^{n_i}\frac{dH_j(r)}{dr}\ u_j^e$$

$$iff\ x \equiv x_1^e + L^e r\ then\ \frac{du(r)}{dx} = \frac{1}{L^e}\sum_{j=1}^{n_i}\frac{dH_j(r)}{dr}\ u_j^e\ \ and\ \ \frac{d^2u(r)}{dx^2} = \frac{1}{(L^e)^2}\sum_{j=1}^{n_i}\frac{d^2H_j(r)}{dr^2}\ u_j^e$$

For a quadratic Lagrange line element (1-----2-----3) with equally spaced physical nodes $J(r) = L^e$ and

$$u(r) = [(1-3r+2r^2)\ \ (4r-4r^2)\ \ (-r+2r^2)]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}^e = [H_1(r)\ \ H_2(r)\ \ H_3(r)]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}^e$$

$$\frac{du(r)}{dr} = [(-3+4r)\ \ (4-8r)\ \ (-1+4r)]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}^e, \text{ and } \frac{du(r)}{dx} = \frac{1}{L^e}\frac{du(r)}{dr}$$

$$\frac{d^2u(r)}{dr^2} = [(4)\ \ (-8)\ \ (4)]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}^e, \text{ and } \frac{d^2u(r)}{dx^2} = \frac{1}{(L^e)^2}\frac{d^2u(r)}{dr^2}$$

Note: $\sum_{j=1}^{n_i} H_j(r) \equiv 1,\ and\ \sum_{j=1}^{n_i}\frac{dH_j(r)}{dr} \equiv 0$