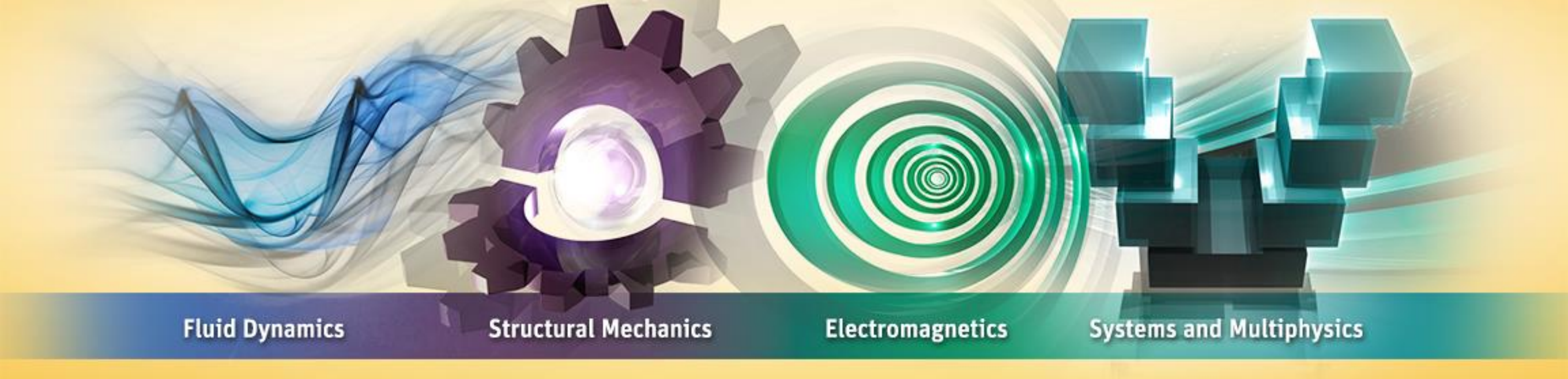
Lecture 8
Modal Analysis

16.0 Release



Introduction to ANSYS Mechanical

Chapter Overview

In this chapter free vibration as well as pre-stressed vibration analyses in Mechanical will be covered.

Chapter Contents:

A. Basics of Free Vibration

B. Theory and Assumption

C. Geometry

D. Contact

E. Solution Setup

F. Modal Results

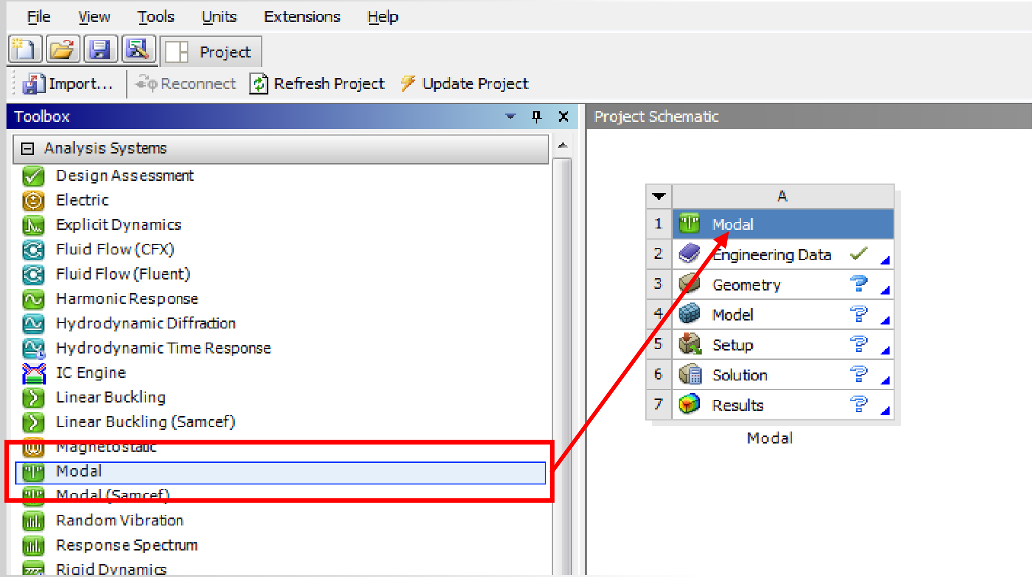
G. Vibration With Prestress

H. Workshop 8.1, Free Vibration – Machine Frame

A. Basics of Free Vibration

The free vibration analysis procedure is very similar to performing a linear static analysis, so not all steps will be covered in detail.

The schematic setup for modal (free vibration) is shown here. Later a prestressed modal setup will be covered.



B. Theory and Assumptions

The linear equation of motion for free, un-damped vibration is

$$[M]\{\ddot{u}\}+[K]\{u\}=\{0\}$$

Assume harmonic motion:

$$\{u\}=\{\phi\}_i \sin(\omega_i t+\theta_i)$$
$$\{\ddot{u}\}=-\omega_i^2\{\phi\}_i \sin(\omega_i t+\theta_i)$$

Substituting $\{u\}$ and $\{\ddot{u}\}$ In the governing equation gives an eigenvalue equation:

$$\left([K]-\omega^2[M]\right)\{\phi_i\}=\{0\}$$

… Theory and Assumptions

As shown on previous slide, for vibration analysis, the natural circular frequencies ω_i and mode shapes ϕ_i are calculated from:

$$\left([K]-\omega_i^2[M]\right)\{\phi_i\}=0$$

Assumptions for modal analysis:

- [K] and [M] are constant:
  - Linear elastic material behavior is assumed
  - Small deflection theory is used, and *no* nonlinearities included
  - [C] is not present, so damping is not included
  - {F} is not present, so no excitation of the structure is assumed
  - The structure can be constrained or unconstrained
- Mode shapes {ϕ} are *relative* values, not absolute

C. Geometry

Modal analysis can employ any type of geometry:

- **Solid bodies, surface bodies and line bodies.**
- **The Point Mass feature can be used:**
  - **A point mass adds mass without additional flexibility to the structure thus reducing the natural frequency (K/M)^0.5.**
- **Material properties: *Young's Modulus, Poisson's Ratio,* and *Density* are required.**

Structural and thermal loads are not available in free vibration:

- **If no supports (or partial) are present, rigid-body modes will occur at or near 0 Hz.**
- **The choice of boundary conditions will affect the mode shapes and frequencies of the part. Carefully consider how the model is constrained.**

D. Contact

Contact regions are available in free vibration analyses however contact behavior will differ for the *nonlinear* contact types:

Contact Type	Modal Analysis		
	Initially Touching	Inside Pinball Region	Outside Pinball Region
Bonded	Bonded	Bonded	Free
No Separation	No Separation	No Separation	Free
Rough	Bonded	Free	Free
Frictionless	No Separation	Free	Free
Frictional	Bonded	Free	Free

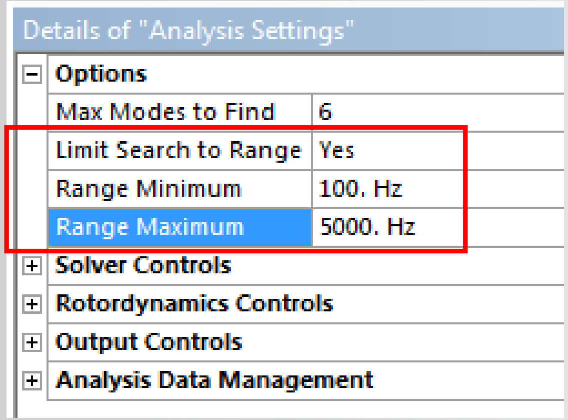
All contact will behave as bonded or no separation in a modal analysis:

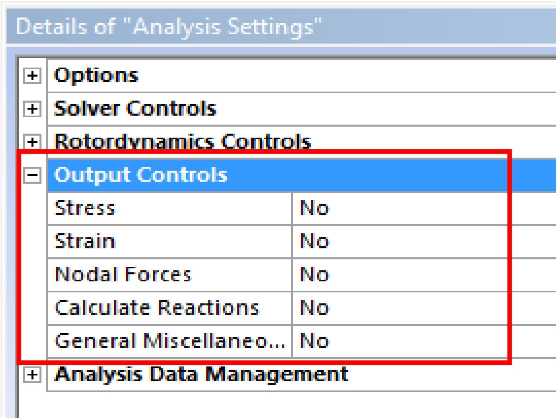
- If a gap is present:
  - Nonlinear contacts will be free (no contact).
  - *Bonded* and *no separation* contact will depend on the pinball size.

E. Solution Setup

Within Mechanical Analysis Settings:

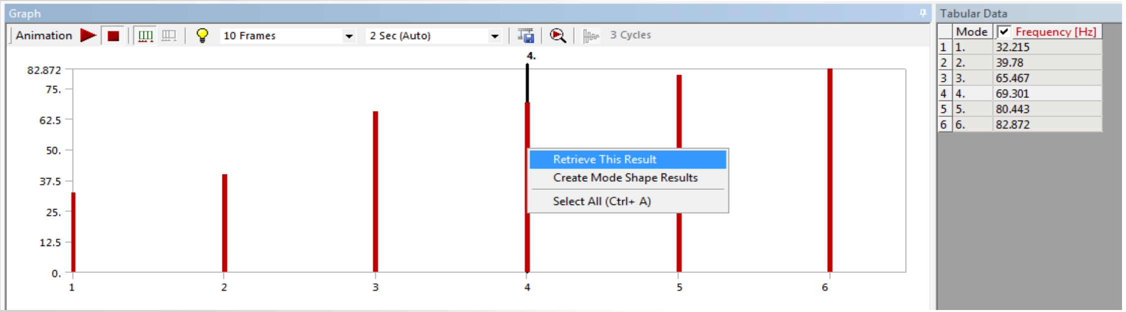
- Specify the number of modes to find (default is 6).
- Optionally specify a frequency search range (defaults from 0Hz to 1e+08Hz).
  – Note: damped modal analysis is covered in the dynamics course.
- Request additional result output if desired.





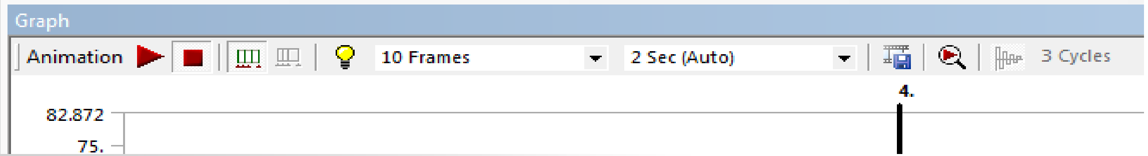
… Solution Setup

When a solution is complete, the solution branch will display a bar chart and table listing frequencies and mode numbers.



RMB to request the modes to be displayed (or select all).

Individual mode shapes can be animated.

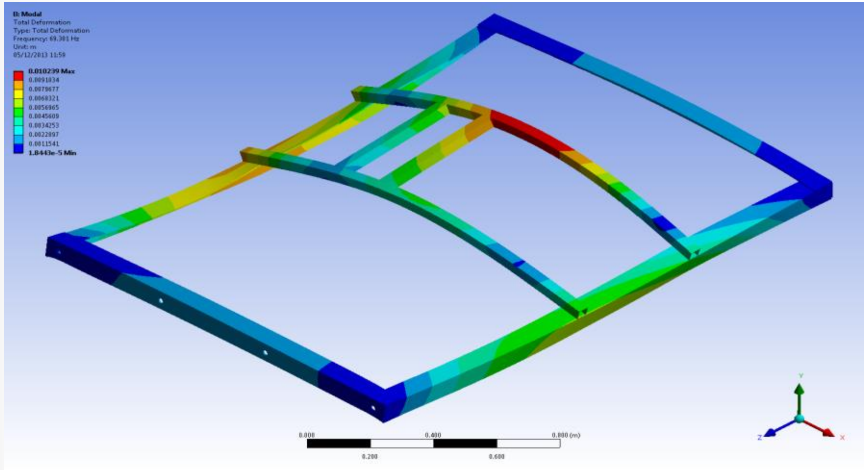


F. Modal Results

Modal Results:

- **Because there is no *excitation* applied to the structure the mode shapes are *relative* values not actual ones.**
- **Mode shape results are mass normalized.**
  - **The same is true for other results (stress, strain, etc.).**

Because a modal result is based on the model's properties and not a particular input, we can interpret where the maximum or minimum results will occur for a particular mode shape but not the actual value.



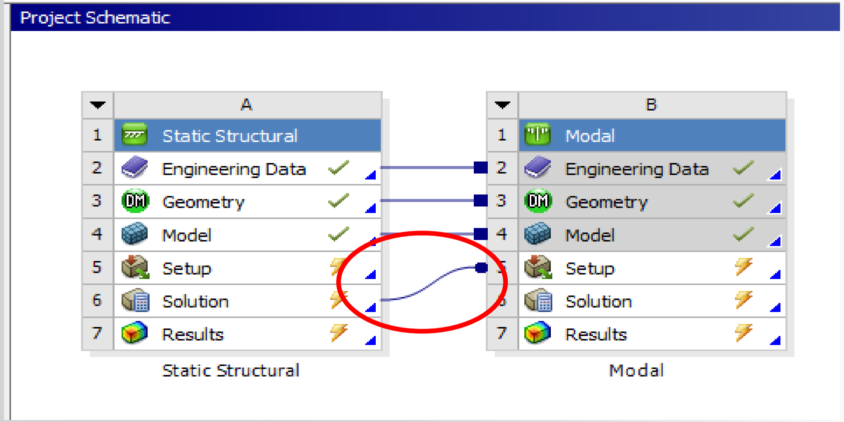
G. Vibration with Pre-Stress

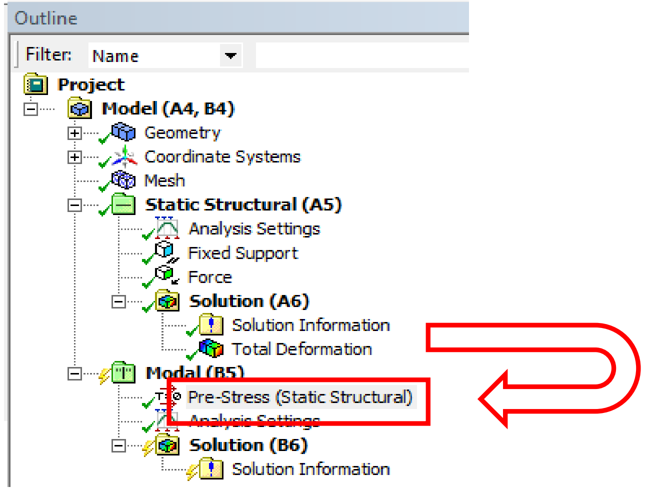
While many prestressed modal examples appear in musical instruments (guitar strings, drum heads, etc.), there are numerous engineering applications where the inclusion of prestress effects are critical.

Note: while prestressing in tension will cause frequencies to increase, compressive states can decrease natural frequencies.

. . . Vibration with Pre-Stress

Setup a pre-stressed modal analysis in the schematic by linking a static structural system to a modal system <u>at the solution level</u>.





- **Notice in the modal branch, the structural analysis result becomes an initial condition.**

. . . Vibration with Pre-Stress

- **The stress state of a structure under influences the modal solution by modifying the stiffness of the structure.**

$$[K]\{x_o\} = \{F\}$$

A linear static analysis is performed

$$[\sigma_o] \rightarrow [S]$$

A stress stiffness matrix is calculated from the structural analysis

$$\left([K+S] - \omega_i^{\,2}[M]\right)\{\phi_i\} = 0$$

The original free vibration equation is modified to include the [S] term

H. Workshop 8.1 – Free Vibration

- **Workshop 8.1 – Free Vibration Analysis**
- **Goal:**
  - **Investigate the vibration characteristics of the machine frame shown here by testing 2 sets of constraints.**