

Plotting the Burzynski Material Failure Criterion in SolidWorks

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This note gives the steps needed to display the Burzynski effective stress in SolidWorks simulation. Like most commercial finite element analysis programs, SolidWorks by default calculates twelve stress components and combinations thereof. Those calculations include the principal stresses (P1, P2, P3 $\leftrightarrow \sigma_1, \sigma_2, \sigma_3$) at a point which define other physically important quantities that relate to various failure criteria, including the hydrostatic pressure

$$p = (\sigma_1 + \sigma_2 + \sigma_3)/3$$

and the von Mises ‘stress’, which actually measures the distortional energy at a point:

$$\sigma_M = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

The von Mises material failure criterion requires that the yield stress in compression and tension are the same and it predicts failure at a point when its value equals or exceeds the yield stress determined in an axial tension test, k_t :

$$\sigma_M = \frac{1}{\sqrt{2}} \sqrt{(k_t - 0)^2 + (k_t - 0)^2 + (0 - 0)^2} = \frac{\sqrt{2}}{\sqrt{2}} k_t = k_t.$$

For many materials the compression yield stress is higher than the tension yield stress. For example, the compressive yield stress of Gray Cast Iron can be three to four times its tensile yield stress. The Burzynski failure criterion [Burzynski 1929 2008, Fras 2010 2013 2014] applies to materials that have different yield stresses in tension and in compression, $k_t \neq k_c$. It states that a material yields when the (deviatoric) volume change energy at a point equals the deviatoric energy in a tension test at the limit of elastic behavior. (A second formulation of the Burzynski criteria can also include the shear stress failure value, k_s). The ratio of the tension to compression yield stresses, $\kappa \equiv k_c/k_t$, is often referred to as the ‘strength difference’. The Burzynski material failure criteria at the point can be determined once the hydrostatic pressure, the von Mises equivalent stress, and the ‘strength difference’ are known. The Burzynski formulation using just the stress difference is:

$$\sigma_B = \left[3p(\kappa - 1) + \sqrt{9p^2(\kappa - 1)^2 + 4\kappa\sigma_M^2} \right] / 2\kappa$$

It also predicts failure at a point when its value equals or exceeds the yield stress in tension. Note that when the material yield stress is the same in tension and compression ($\kappa = 1$) then the Burzynski criterion reduces to the von Mises criterion at impending failure:

$$\sigma_B(@\kappa = 1) \equiv \sigma_M = k_t.$$

The most common yield criterion for ductile metals is the von Mises criterion and it is the default stress display in most commercial FEA codes, including SolidWorks. SolidWorks does not specifically automatically offer a plot of the Burzynski failure criterion, but it does allow user defined stress plots (after the SolidWorks 2017 release) by providing a ‘**Results Equation**’ plot in the **Results** section, as shown in Figure 3.

The standard SolidWorks stress data can be substituted into the SolidWorks ‘stress editor’ to define the Burzynski failure criterion plot, as well as other criteria. The hydrostatic pressure is one third of the tri-axial stress at the point, called TRI in SolidWorks and is identified as “TRI: Triaxial Stress (P1+P2+P3)” in the editor; so $3p = TRI$. The von Mises stress is called VON in SolidWorks and in the editor is denoted as “VON:

von Mises Stress”. In general, the SolidWorks default editor terminology for the Burzyski effective stress becomes:

$$\sigma_B = \left[(\kappa - 1) TRI + \sqrt{(\kappa - 1)^2 TRI^2 + 4\kappa VON^2} \right] / 2\kappa$$

The numerical value of κ must be known first before the editor is invoked. The compressive and tensile yield stresses of hundreds of materials are given at <http://www.matweb.com> and the strength difference of the material can be computed from those data.

To aid the reader in using the editor the above general expression will re-written using three constants, defined below, as:

$$\sigma_B = 2 * [A * TRI + \sqrt{B * TRI^2 + C * VON^2}] / C$$

with	κ	$A = (\kappa - 1)$	$B = (\kappa - 1)^2$	$C = 4\kappa$
	0.7	-0.3	0.09	2.8
	0.8	-0.2	0.04	3.2
	0.9	-0.1	0.01	3.6
	1.0	0.0	0.00	4.0
	1.1	0.1	0.01	4.4
	1.2	0.2	0.04	4.8
	1.3	0.3	0.09	5.2
	1.4	0.4	0.16	5.6
	1.5	0.5	0.25	6.0
	1.6	0.6	0.36	6.4
	1.7	0.7	0.49	6.8
	1.8	0.8	0.64	7.2
	1.9	0.9	0.81	7.6
	2.0	1.0	1.00	8.0
	2.5	1.5	2.25	10.
	3.0	2.0	4.00	12.
	4.0	3.0	9.00	16.

In the above notation, the text defining the Burzyski stress value to plot appears in the plot definition window as

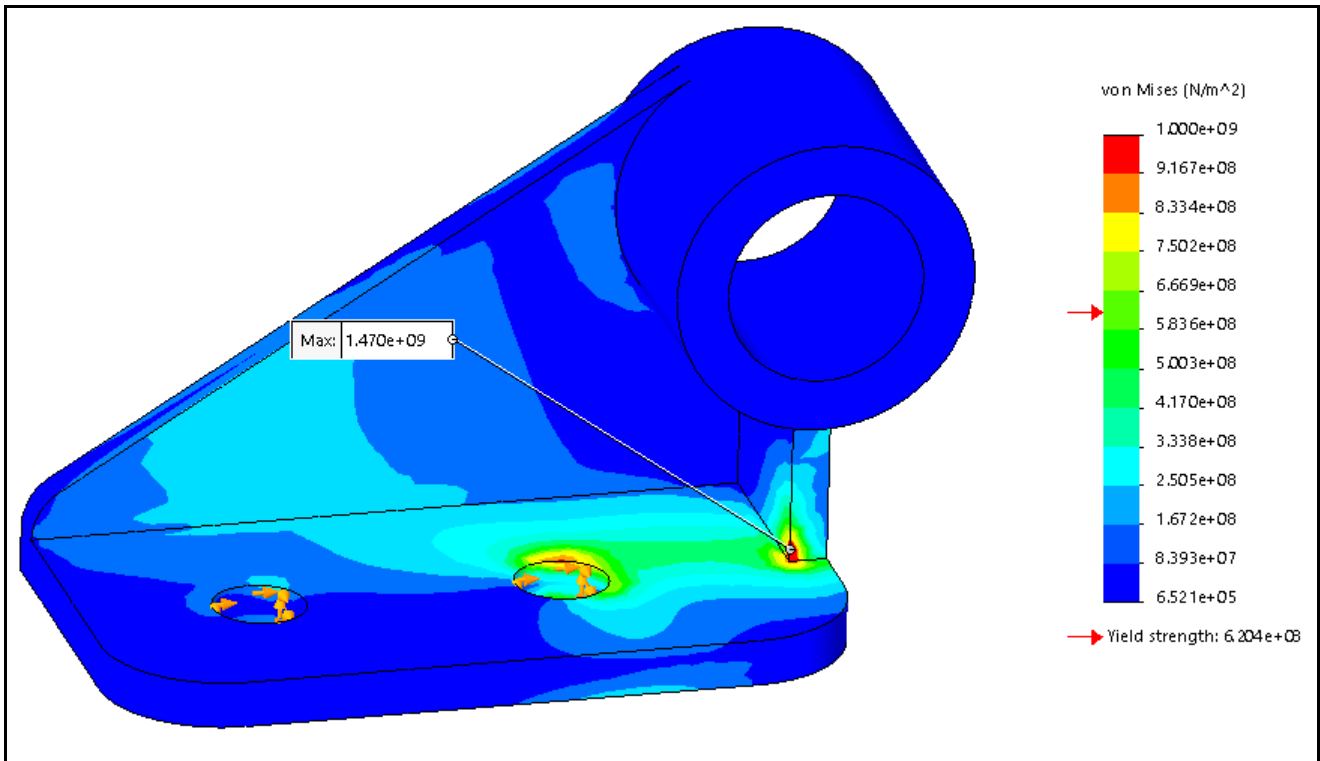
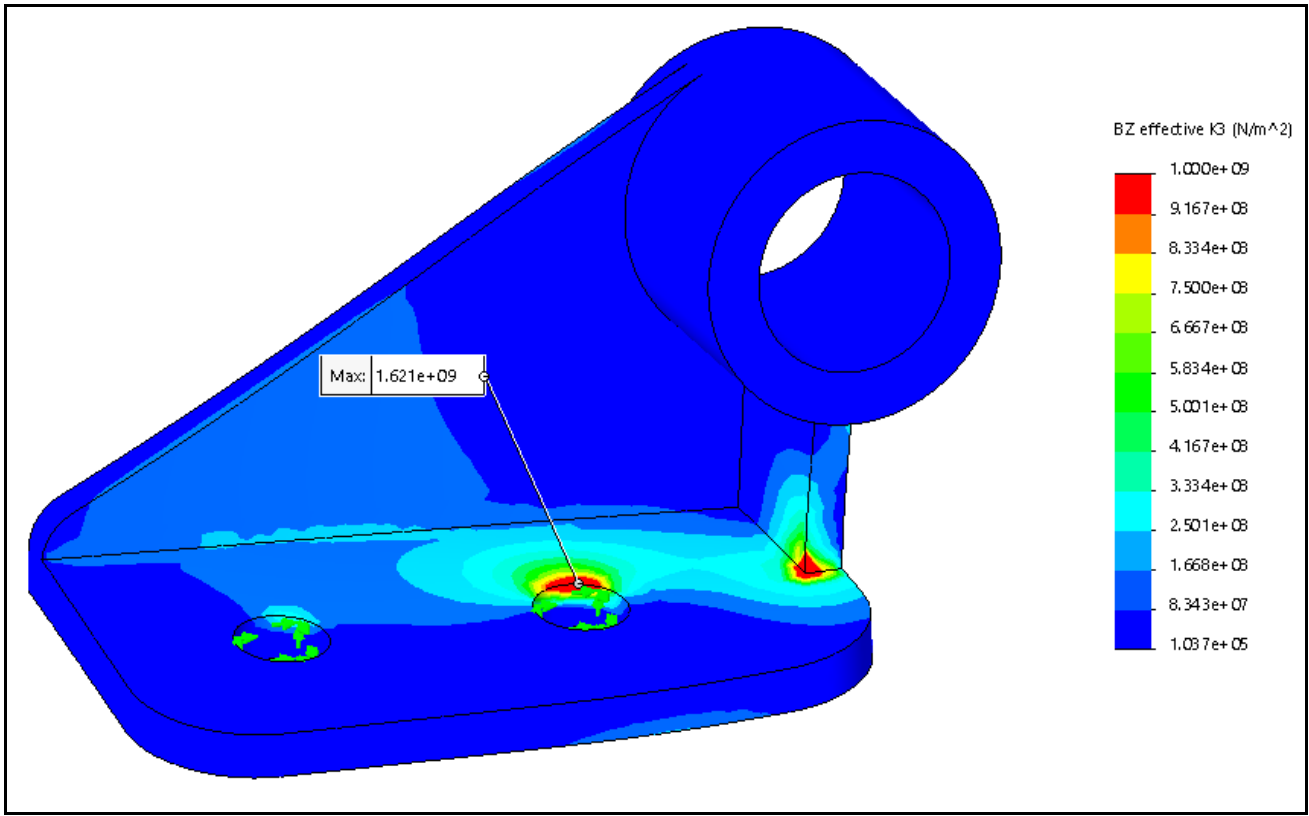
$$2*(A*"TRI: Triaxial Stress(P1+P2+P3)" + \text{sqr}(B*"TRI: Triaxial Stress(P1+P2+P3)"*"TRI: Triaxial Stress(P1+P2+P3)" + C*"VON: von Mises Stress"*"VON: von Mises Stress"))/C$$

For example, with a material with a compressive yield stress twice that of its tension value so $\kappa = 2$ substituting into the general equation above gives

$$2*(1*"TRI: Triaxial Stress(P1+P2+P3)" + \text{sqr}(1*"TRI: Triaxial Stress(P1+P2+P3)"*"TRI: Triaxial Stress(P1+P2+P3)" + 8*"VON: von Mises Stress"*"VON: von Mises Stress"))/8$$

Finally, to define and display the Burzyski effective stress in SolidWorks Simulation select **Results → Results Equation Plot → Definition → Edit Equation**. Set your usual options, like **Settings → Fringe Options → Discrete**, etc. Then select and type in the above specific equation into the edit box → OK. Then select the stress **Units** and edit the ‘**Legend title**’ text to be “Burzyski (units)”. Select any other standard user default options, such as **Discrete** contours and plot the criterion.

When applied to the first SolidWorks stress tutorial, and where the material is changed to cast iron with $\kappa = 3$ the two comparison plots show that the Burzynski effective stress predicts yielding (failure) at a 10.1 % lower applied pressure than the von Mises effective stress does, and it predicts at a different yield location on the part:



References

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2. T. Fras, Z.L. Kowalewski, R.B. Pecherski, A. Rusinek, "Applications of Burzynski failure criteria, Part 1, Isotropic materials with asymmetry of elastic range", Engng. Trans., 58, 1–2, 3–13, 2010.
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Appendix: Stress editor panels

