HW 8 Mech 417/517 Beams: Posted 2/19/19 due 2/26/19

1. The class notes section posted the solution of a cantilever beam with a triangular line load. The solution gave the exact analytic solution at the nodes:

$$\boldsymbol{v^e} = [v_1 \quad \theta_1 \quad v_2 \quad \theta_2]^T = \frac{wL^3}{EI}\begin{bmatrix}\frac{L}{30} & \frac{-1}{24} & 0 & 0\end{bmatrix}^T, \text{Note load } W = wL/2$$

Since each classic cubic beam has four constants, the deflection of any point along the length is approximated by a cubic polynomial (for $r = x/L$):

$$v(x) = v_1(1 - 3r^2 + 2r^3) + \theta_1(r - 2r^2 + r^3)L + v_2(3r^2 - 2r^3) + \theta_2(r^3 - r^2)L$$

The consistent finite element theory for the cubic beam approximation the moment and shear are linear and constant along the beam length, respectively. Specifically, they are:

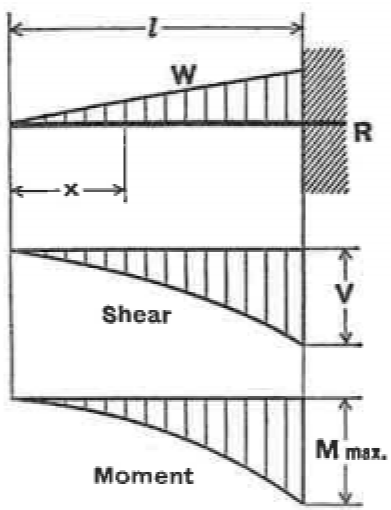
$$M(x) = [v_1(12r - 6) + \theta_1(6r - 4)L + v_2(6 - 12r) + \theta_2(6r - 2)L]/L^2$$

$$V(x) = [v_1(12) + \theta_1(6)L + v_2(-12) + \theta_2(6)L]/L^3.$$

The exact beam solution from the AISC Handbook is given in the figure below. Plot the single element cubic beam solution for the moment and shear along with the exact values.

Mech 517: Also plot the approximate and exact deflections.

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



Equivalent Tabular Load	$= \frac{8}{3}W$
R = V	$= W$
V_x	$= W\frac{x^2}{l^2}$
M max. (at fixed end)	$= \frac{Wl}{3}$
M_x	$= \frac{Wx^3}{3l^2}$
Δmax. (at free end)	$= \frac{Wl^3}{15EI}$
Δx	$= \frac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

Posted 2/19/19 due 2/28/19

2. For the class notes section posted the solution of a cantilever beam with a triangular line load use the last two original matrix rows to determine the force and moment reactions at the support wall. Draw a free body diagram of the system. Use static force and moment equilibrium equations to verify that the reactions are exact.

$$\frac{EI}{L^3}\left[\begin{array}{cc|cc} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ \hline -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{array}\right]\left\{\begin{array}{c} v_1 \\ \theta_1 \\ \hline v_2 \\ \theta_2 \end{array}\right\} = \frac{WL}{2}\left\{\begin{array}{c} 3/10 \\ L/15 \\ \hline 7/10 \\ -L/10 \end{array}\right\} + \left\{\begin{array}{c} 0 \\ 0 \\ F_2 \\ M_2 \end{array}\right\}$$