Numerical integration: $\int_{\boxdot} \boldsymbol{F}(r)\ d\boxdot \approx \sum_{q=1}^{n_q} \boldsymbol{F}(r_q)\, w_q$

$$\int_{\Omega} \boldsymbol{F}(r)\, d\Omega = \int_{\boxdot} \boldsymbol{F}(r)\, |\boldsymbol{J}^e|\ d\boxdot \approx \sum_{q=1}^{n_q} \boldsymbol{F}(r_q)\, |\boldsymbol{J}^e(r_q)| w_q$$

Number of points for exact polynomial integration: $Degree \le (2n_q - 1)$

Gaussian 1-D one-point line rule data, in unit coordinates and natural coordinates:

$$r_1 = 0.5,\ \ w_1 = 1\ \ ;\ \ \ a_1 = 0,\ w_1 = 2$$

Gaussian 1-D two-point line rule data, in unit coordinates and natural coordinates:

$$r_1 = (1 - 1/\sqrt{3})/2 = 0.21132, r_2 = (1 + 1/\sqrt{3})/2 = 0.78867,$$
$$w_1 = w_2 = 0.50000$$
$$a_1 = -1/\sqrt{3} = -0.57735,\ a_2 = 1/\sqrt{3} = 0.57735,$$
$$w_1 = w_2 = 1.00000$$

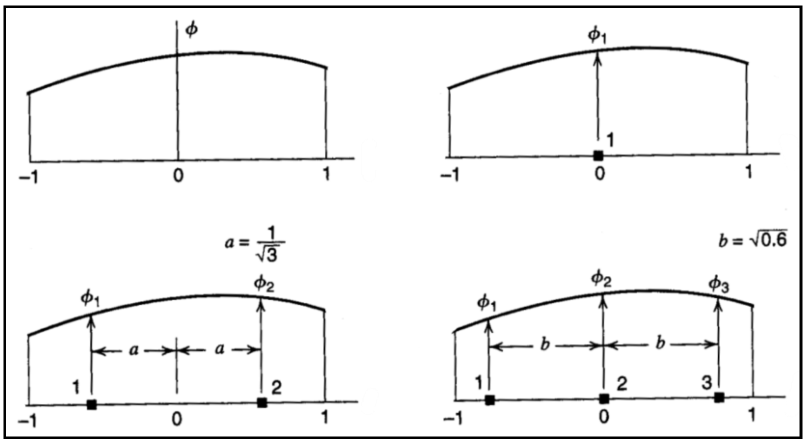

Figure 3.1-1 Natural 1, 2, and 3 point quadrature points, ξ, on a line

Mech 417/517 Homework

HW 2, 1/25/18 due 1/30/18

1. Evaluate the ‘measure’ of the unit coordinate space by numerical integration:

$$I_r = \int_{\boxdot} d\boxdot = \int_0^1 dr \quad \left(\int_0^1 1\ dr\right)$$

2. Evaluate the ‘measure’ of the natural coordinate space by numerical integration:

$$I_a = \int_{\boxdot} d\boxdot = \int_{-1}^1 da \quad \left(\int_{-1}^1 1\ da\right)$$

HW 3, 1/25/18 due 2/1/18

1. Evaluate the integral of the interpolation functions of a quadratic line element in unit coordinates where $\boldsymbol{H}(r) = [(1 - 3r + 2r^2) \quad (4r - 4r^2) \quad (-r + 2r^2)]$:

$$\boldsymbol{I_r} = \int_{\boxdot} \boldsymbol{H}\ d\boxdot = \int_0^1 \boldsymbol{H}\ dr$$

2. For uniform meshes and constant properties, the symmetric square matrix $\boldsymbol{m}$ often appears where $\boldsymbol{m} = \int_0^1 \boldsymbol{H}(r)^{\boldsymbol{T}}\, \boldsymbol{H}(r)\ dr$ of size $n_n \times n_n$ for a line element with n_n nodes. Determine the

polynomial degree of the integrand and the number of Gaussian quadrature points needed to integrate that matrix for a cubic line element, $n_n = 4$, where

$$\boldsymbol{H}(r) = \frac{1}{2}[(2 - 11r + 18r^2 - 9r^3) \quad (18r - 45r^2 + 27r^3) \ldots$$
$$(-9r + 36r^2 - 27r^3) \quad (2r - 9r^2 + 9r^3)] = [H_1 \ \ H_2 \ \ H_3 \ \ H_4]$$

3. For a cubic line element evaluate the scalar sum $s = \sum_k H_k\,(r)$.

HW 4, 2/1/18 due 2/6/18

1. Resolve Example 8.1-2 using a linear element with $\boldsymbol{H}(r) = [(1 - r) \ \ r]$.

2. Resolve Example 8.1-3 using a linear element with $\boldsymbol{H}(r) = [(1 - r) \ \ r]$.

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3. In addition to all of the above assignments, use numerical integration to form the matrix $\boldsymbol{m} = \int_0^1 \boldsymbol{H}(r)^{\boldsymbol{T}}\, \boldsymbol{H}(r)\, dr$ for a cubic line element.