Open Book, Open Notes, Open Web, Time Limited Test1 for Mech417/517

JEA19417

Available beginning Feb. 17, 2019 with hardcopy solution due 5pm Feb. 25
at the ME "In-Box" wall slot near room ME 109.

Cover Page

Instructions

You must work alone on this test. If you are not familiar with the Rice Honor system, then review it before proceeding. Once you go beyond this cover page, you have a maximum of two hours to work on this test, excluding an optional 10-minute break that prohibits consulting reference materials.

This is an open book, open notes and open course web page test.

There are multiple choice, and simple calculations among the questions. There are 10 questions for Mech 417 and 14 for Mech 517.

Mech 417/517, Finite Element Analysis, Prof. Akin, Spring 2019

Name _____ **Begin Time** _____ **End Time** _____

1. What are the essential and non-essential boundary conditions for a fourth order ordinary differential equation?

2. The inertia of the unit line domain, with respect to point r = 0, is $I_{ss} = \int_0^1 r^2 \, dr$. Evaluate this quantity a) by exact integration, b) numerical integration.

3. The inertia of a rigid straight bar, with respect to x=0, is $I_{yy} = \int_{x_1}^{x_2} x^2 \, dx$. Interpolate the position as $x(r) = \boldsymbol{H}(r) \, \boldsymbol{x^e}$ where the end coordinates are $\boldsymbol{x^e} = [x_1 \;\; x_2]^T$.
   A) Evaluate the Jacobian of the geometric mapping.
   B) Write the scalar integrand as a product of matrices, including $\boldsymbol{x^e}$.
   C) Evaluate the matrix integral using the parametric space, $0 \leq r \leq 1$. Leave it in matrix form and DO NOT expand the result back to the scalar form involving x_1 and x_2.
   D) What is the polynomial degree of the square matrix integrand?
   E) For a square matrix integrand of polynomial degree of four how many Gaussian integration points are needed to exactly evaluate the integration?
   F) Evaluate the inverse Jacobian of the geometric mapping.

4. What are the basic concepts for isoparametric elements?

5. The mapping from a parametric space to physical space is done with the ____

a) determinant of the Jacobian	b) inverse of the Jacobian matrix
c) Jacobian matrix	d) interpolation functions
e) derivatives of interpolation functions	f) parametric derivatives

6. The physical differential volume is related to the parametric differential volume by the _____

a) determinant of the Jacobian	b) inverse of the Jacobian matrix
c) Jacobian matrix	d) interpolation functions
e) derivatives of interpolation functions	f) parametric derivatives

7. The physical partial derivative of a quantity is related to the parametric partial derivative of a quantity multiplied by the _____

a) determinant of the Jacobian
b) inverse of the Jacobian matrix
c) Jacobian matrix
d) interpolation functions

8. The singular matrix equilibrium form can be partitioned into the form

$$\begin{bmatrix} \mathbf{S_{uu}} & \mathbf{S_{uk}} \\ \mathbf{S_{ku}} & \mathbf{S_{kk}} \end{bmatrix} \begin{Bmatrix} \mathbf{u_u} \\ \mathbf{u_k} \end{Bmatrix} = \begin{Bmatrix} \mathbf{c_u} \\ \mathbf{c_k} + \mathbf{r_k} \end{Bmatrix}$$

where the $\mathbf{u_u}$ are the independent unknowns, and the $\mathbf{u_k}$ are known from the given boundary conditions, the $\mathbf{r_k}$ are the reactions needed to maintain those boundary conditions, and $\mathbf{c_u}$ and $\mathbf{c_u}$ are known resultant source terms. All of the terms in the square matrix are known.

A) Write the matrix operations needed to compute the independent unknowns.
B) Write the matrix operations needed to compute the system reaction quantities.

9. The interpolation equations for a three node line element are $\boldsymbol{H} = [(r-1)(2r-1),\ 4r(1-r),\ r(2r-1)]$. A distorted three node line element has node x-coordinates of $\boldsymbol{x}^{e^T} = [0 \quad 0.3L \quad L\ ]$. Evaluate $\partial x/\partial r$ for the element.

10. Provide a question and answer about finite element analysis.

MECH 517 (or 417 Bonus)

11. According to Tong's Theorem, what restriction on the interpolation functions is required to assure that the finite element solution will be exact at the nodes (but maybe not elsewhere in the element)?

12. The consistent advection matrix for any element is $\mathbf{U}^{\mathrm{e}} = \int_{\mathrm{L}} \mathbf{H}^{\mathrm{e}\mathrm{T}} \mathrm{v} \frac{d\mathbf{H}^{\mathrm{e}}}{dx}\, \mathrm{dx}$. For a four-node line element (L4) in the unit space, 0 <= r <= 1, explain how many quadrature points would be required to integrate this matrix exactly for equally spaced physical nodes if the velocity, v, is constant.

13. Exactly evaluate the advection matrix for a two-node linear line element.

14. Most non-essential boundary conditions produce only a boundary column matrix resultant term. However, mixed (or Robin) boundary conditions are coupled to the unknown dof and thus create a boundary square matrix. Heat convection at a surface is a mixed boundary condition. What are the matrix definitions of its boundary convection square matrix and convection source vector?