MECH 517 – Finite Element Analysis

Homework 3

1) Evaluate the 'measure' of the unit coordinate space by numerical integration:

$$I_r = \int_{\square} d\square = \int_0^1 dr \quad \left(\int_0^1 1\, dr\right)$$

The exact integral is:

$I_r = r|_0^1 \rightarrow I_r = 1$

Using numerical integration:

$$\int_{\square} F(r) d\square \approx \sum_{q=1}^{n_q} F(r_q)\, w_q$$

$F(r) = 1$

Number of points for exact polynomial integration: $Degree \leq (2n_q - 1)$

$Degree = 1 \rightarrow n_q = 1$

$r_1 = 0.5 \qquad w_1 = 1$

$$\sum^{1} F(r_1) w_1 \rightarrow F(0.5) * 1 \rightarrow 1 * 1 = 1$$

2) Evaluate the 'measure' of the natural coordinate space by numerical integration:

$$I_a = \int_{\square} d\square = \int_{-1}^1 da \quad \left(\int_{-1}^1 1\, da\right)$$

The exact integral is:

$I_a = a|_{-1}^1 \rightarrow I_a = 2$

Using numerical integration:

$$\int_{\square} F(a)d\square \approx \sum_{q=1}^{n_q} F(a_q)\, w_q$$

$F(a) = 1$

Number of points for exact polynomial integration: $Degree \leq (2n_q - 1)$

$Degree = 1 \rightarrow n_q = 1$

$a_1 = 0 \qquad w_1 = 2$

$$\sum^{1} F(a_1)w_1 \rightarrow F(0) * 2 \rightarrow 1 * 2 = 2$$