MECH 517 – Finite Element Analysis

Homework 4

1) **Evaluate the integral of the interpolation functions of a quadratic line element in unit coordinates where $H(r) = [(1-3r+2r^2) \quad (4r-4r^2) \quad (-r+2r^2)]$**

$$I_r = \int_{\boxdot} H \, d\boxdot = \int_0^1 H \, dr$$

$$I_r = \int_0^1 H(r)\, dr \rightarrow \begin{bmatrix} \int_0^1 (1-3r+2r^2)dr \\ \int_0^1 (4r-4r^2)dr \\ \int_0^1 (-r+2r^2)dr \end{bmatrix}^T \rightarrow \begin{bmatrix} r - \frac{3}{2}r^2 + \frac{2}{3}r^3 \Big|_0^1 \\ 2r^2 - \frac{4}{3}r^3 \Big|_0^1 \\ -\frac{r^2}{2} + \frac{2}{3}r^3 \Big|_0^1 \end{bmatrix}^T \rightarrow \begin{bmatrix} 1 - \frac{3}{2} + \frac{2}{3} \\ 2 - \frac{4}{3} \\ -\frac{1}{2} + \frac{2}{3} \end{bmatrix}^T$$

$$\rightarrow \begin{bmatrix} \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix}$$

2) **For uniform meshes and constant properties, the symmetric square matrix m often appears where**

$$m = \int_0^1 H(r)^T H(r)\, dr$$

Of size $n_n \times n_n$ for a line element with n_n nodes. Determine the polynomial degree of the integrand and the number of Gauss quadrature points needed to integrate that matrix for a cubic line element, $n_n = 4$ where

$$H(r) = \frac{1}{2}\begin{bmatrix} (2-11r+18r^2-9r^3) \\ (18r-45r^2+27r^3) \\ (-9r+36r^2-27r^3) \\ (2r-9r^2+9r^3) \end{bmatrix}^T = [H_1 \quad H_2 \quad H_3 \quad H_4]$$

Based on the element type and number of nodes (Lagrange L4) and the highest degree in $H(r)$, the polynomial degree of the integrand is 3.

X $3+3=6$

The number of quadrature points is based on:

$Degree \le (2n_q - 1) \rightarrow 3+3 \le (2n_q - 1) \rightarrow 7 \le 2n_q \rightarrow 3.5 \le n_q$

Therefore, the number of quadrature points (n_q) should be 4. only after integration

3) For a cubic line element evaluate the scalar sum $s = \sum_k H_k(r)$.

$$s = \sum_k H_k(r)$$

$$s = \sum \frac{1}{2}[H_1 \quad H_2 \quad H_3 \quad H_4]$$

$$s = \frac{1}{2}[2 - 11r + 18r^2 - 9r^3 + 18r - 45r^2 + 27r^3 - 9r + 36r^2 - 27r^3 + 2r - 9r^2 + 9r^3]$$

$$s = 1 + (0)r + (0)r^2 + (0)r^3$$