MECH 517 Homework 8

Problem 1:
Plot the single element cubic beam solution for the displacement, moment, and shear for a cantilever beam with triangular line loading, along with the exact values.

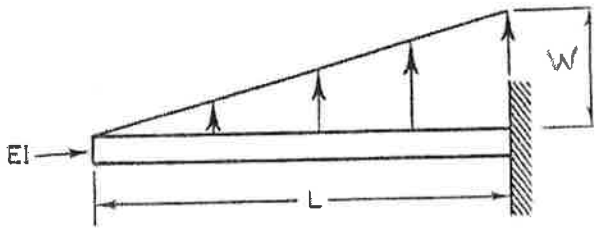

Figure 1: Cantilever Beam with Triangular Line Loading

Solution:
I set the values of W, E, I, and L arbitrarily to:

$$\begin{aligned} W &= 1 \ N \\ E &= 1 \ Pa \\ I &= 1 \ m^4 \\ L &= 1 \ m \end{aligned}$$

For the finite element solutions, I used the following equations:

$$\begin{aligned} v(x) &= v_1(1 - 3r^2 + 2r^3) + \theta_1(r - 2r^2 + r^3)L + v_2(3r^2 - 2r^3) + \theta_2(r^3 - r^2)L \\ M(x) &= [v_1(12r - 6) + \theta_1(6r - 4)L + v_2(6 - 12r) + \theta_2(6r - 2)L]/L^2 \\ V(x) &= [v_1(12) + \theta_1(6)L + v_2(-12) + \theta_2(6)L]/L^3 \end{aligned}$$

While the above equations will only approximate the solution, it is important to note that the shear and moment functions are constant and linear, respectively. This being the case, they will not accurately represent the nonlinear shape of the exact shear and moment diagrams. For the exact solutions, I derived the following equations based on the class notes:

$$v(x)_e = \frac{WL^4}{120EI}(4 - 5(x/L) + (x/L)^5)$$

$$M(x)_e = v(x)_e'' = \frac{WL^4}{6EI}\left(\frac{x}{L}\right)^3$$

$$V(x)_e = v(x)_e''' = \frac{WL^4}{2EI}\left(\frac{x}{L}\right)^2$$

The plots for the displacement, shear, and moment for both the exact solution and the cubic element approximation have been included below:

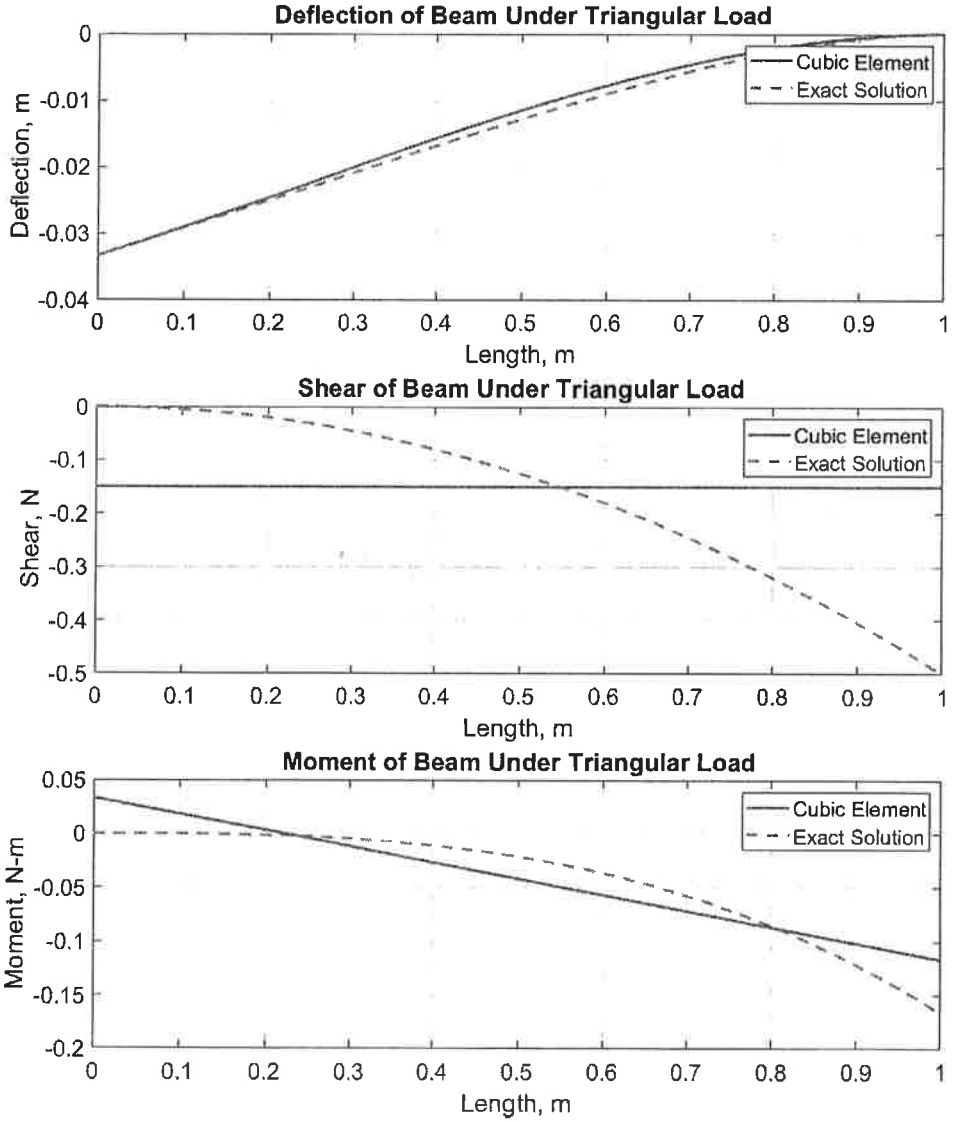

Figure 2: Cantilever Beam with Triangular Line Loading

The MATLAB code used to derive these values has been appended on the next page.