MECH 517 Homework 8 Part 2

Problem:
a) Determine the force and moment reactions based on the matrix equation below.
b) Draw a free body diagram of the system and use static force & moment balance equations to verify that the reactions are exact.

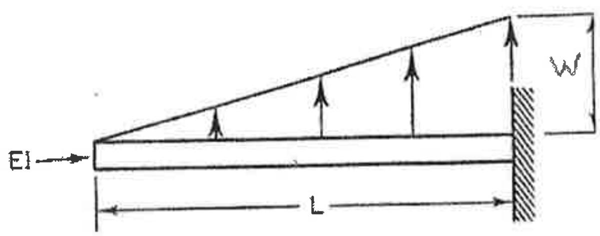

Figure 1: Cantilever Beam with Triangular Line Loading

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \frac{WL}{2}\begin{Bmatrix} 3/10 \\ L/15 \\ 7/10 \\ -L/10 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ F_2 \\ M_2 \end{Bmatrix}$$

Solution: reference exact
a) The ~~first~~ step is to find exact values for v_1 and θ_1 as a function of W, L, E, and I. as a future check

$$v_1 = v(x) = \frac{WL^4}{120EI}\left[4 - \frac{5x}{L} + \left(\frac{x}{L}\right)^5\right]$$

$$\theta_1 = v'(x) = \frac{WL^4}{120EI}\left[\frac{5}{L} + 5\left(\frac{x}{L}\right)^4\right]$$

$$\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \frac{WL^3}{EI}\begin{Bmatrix} L/30 \\ -1/24 \\ 0 \\ 0 \end{Bmatrix}$$

solving the FE gives the same exact values (missing)

Now that these terms are known, we can substitute them back into the original matrix and solve for F_2 and M_2. Since these terms only appear in the last two matrix rows, we only need to solve these sets of equations to determine these values.

assigned, missing

$$\frac{EI}{L^3}\begin{bmatrix} -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \frac{WL}{2}\begin{Bmatrix} 7/10 \\ -L/10 \end{Bmatrix} + \begin{Bmatrix} F_2 \\ M_2 \end{Bmatrix}$$

$$\frac{EI}{L^3}\left(\frac{-12WL^4}{30EI} + \frac{6WL^4}{24EI} + 12(0) - 6L(0)\right) = \frac{7WL}{20} + F_2$$

$$\frac{-8WL}{20} + \frac{5WL}{20} - \frac{7WL}{20} = F_2$$

$$\boxed{\mathbf{F_2 = \frac{-WL}{2}}}$$

$$\frac{EI}{L^3}\left(\frac{6WL^5}{30EI} - \frac{2WL^5}{24EI} - 6L(0) + 4L^2(0)\right) = \frac{-WL^2}{20} + M_2$$

$$\frac{6WL^2}{30} - \frac{2WL^2}{24} + \frac{WL^2}{20} = M_2$$

$$\boxed{\mathbf{M_2 = \frac{WL^2}{6}}}$$

b) These results can be compared to the results obtained from a simple free-body diagram of the system and the reactions resulting from the force & moment balance equations as shown below:

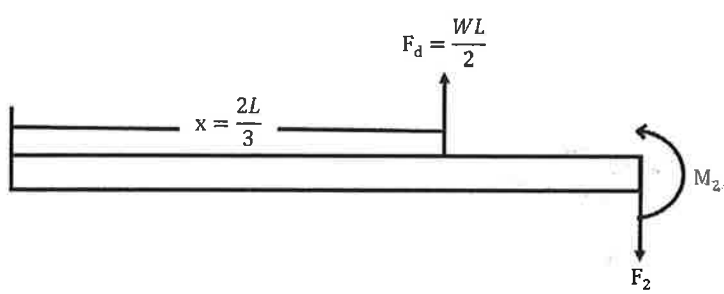

Figure 2: Free Body Diagram of Beam with Triangle Loading

$$\sum F_y = 0: \quad \frac{WL}{2} + F_2 = 0 \Rightarrow \boxed{\mathbf{F_2 = \frac{-WL}{2}}}$$

$$\sum M = 0: \quad M_2 - \left(\frac{WL}{2}\right)\left(\frac{L}{3}\right) = 0 \Rightarrow \boxed{\mathbf{M_2 = \frac{-WL^2}{6}}}$$

The above solutions match exactly with those obtained using the matrix equations used in (a).