MECH 517 Homework 10

Problem:

a) As sketched below, a unit parametric triangle is mapped to the flat x-y plane and has a normal applied pressure acting on it. The geometric and pressure data are:

Node	x(m)	y(m)	p(N/m²)
1	0	0	40
2	4	0.5	34
3	2	5	46

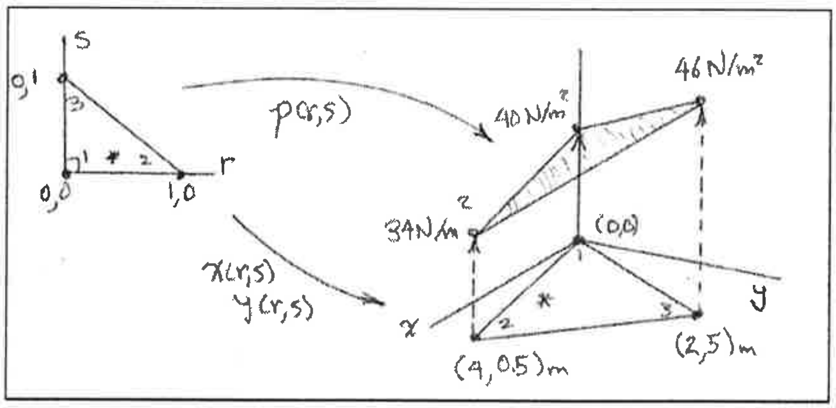

Figure 1: Geometric and pressure data

At the parametric point (r=0.37, s=0.24), find:

a. The coordinates on the plane
b. The pressure at the point
c. The 2x2 Jacobian matrix
d. The determinant of the Jacobian matrix
e. The inverse of the Jacobian matrix
f. The local parametric pressure gradient
g. The physical pressure gradient
h. 517 - the physical pressure gradient at parametric point (0.4,0.4)

Solution:

a) The coordinates on the plane can be calculated using the following equation.

$$[x(r,s), y(r,s)] = H(r,s)[x^e y^e] = \begin{bmatrix}(1-r-s) & r & s\end{bmatrix}\begin{bmatrix}0 & 0\\4 & 0.5\\2 & 5\end{bmatrix}$$

$$\boxed{(x,y) = (1.96, 1.385)m}$$

b) The pressure at the point can be found in a similar manner.

$$P(x,y) = H(r,s)[P] = \begin{bmatrix}(1-r-s) & r & s\end{bmatrix}\begin{bmatrix}40\\34\\46\end{bmatrix}$$

$$\boxed{P = 39.22N/m^2}$$

c) The 2x2 Jacobian matrix can be found as follows:

$$J = H'(r,s)R^e = \begin{bmatrix}-1 & 1 & 0\\-1 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1 & y_1\\x_2 & y_2\\x_3 & y_3\end{bmatrix}$$

$$\boxed{J^e = \begin{bmatrix}4 & 0.5\\2 & 5\end{bmatrix} m}$$

d) The determinant of the Jacobian matrix can be found as follows:

$$|J| = (x_2 - x_1)^e(y_3 - y_1)^e - (x_3 - x_1)^3(y_2 - y_1)^e = 2A^e$$
$$|J| = (4-0)(5-0) - (4-2)(.5-0)$$

$$\boxed{|J| = 19m^2}$$

e) The inverse of the Jacobian matrix can be found as follows:

$$J^{-1} = \frac{1}{det(J)}\begin{bmatrix}(y_3 - y_1) & -(y_2 - y_1)\\-(x_3 - x_1) & (x_2 - x_1)\end{bmatrix}$$

$$J^{-1} = \frac{1}{19}\begin{bmatrix}(5-0) & -(0.5-0)\\-(2-0) & (4-0)\end{bmatrix}$$

$$\boxed{J^{-1} = \begin{bmatrix}0.2632 & -0.0263\\-0.1053 & 0.2105\end{bmatrix} m^{-1}}$$

f) The local parametric pressure gradient can be found as follows:

$$\nabla P(r,s) = H'(r,s)P = \begin{bmatrix}-1 & 1 & 0\\-1 & 0 & 1\end{bmatrix}\begin{bmatrix}40\\34\\46\end{bmatrix}$$

$$\boxed{\nabla P(r,s) = \begin{bmatrix}-6\\6\end{bmatrix} N/m^2}$$

g) The physical pressure gradient can be related back to the parametric pressure gradient using the inverse of the Jacobian as shown:

$$\nabla P(x,y) = J^{-1}\nabla P(rs) = \begin{bmatrix} 0.2632 & -0.0263 \\ -0.1053 & 0.2105 \end{bmatrix} \begin{bmatrix} -6 \\ 6 \end{bmatrix}$$

$$\boxed{\nabla P(x,y) = \begin{bmatrix} -1.7368 \\ 1.8947 \end{bmatrix} N/m^3}$$

g) The physical pressure gradient at parametric point (0.4,0.4) can be calculated following the same process as detailed above. The pressure gradient at this point is found to be the same as at the previous point:

$$\boxed{\nabla P(x,y) = \begin{bmatrix} -1.7368 \\ 1.8947 \end{bmatrix} N/m^3}$$

The MATLAB code used to calculate these values has been appended on the next page.